

TESTING GENERAL RELATIVITY WITH SATELLITE LASER RANGING: RECENT DEVELOPMENTS

L. Iorio ¹

¹*Dipartimento di Fisica dell'Università di Bari, Via Amendola 173, 70126, Bari, Italy*

ABSTRACT

In this paper the most recent developments in testing General Relativity in the gravitational field of Earth with the technique of Satellite Laser Ranging are presented. In particular, we concentrate our attention on some gravitoelectric and gravitomagnetic post-Newtonian orbital effects on the motion of a test body in the external field of a central mass. Following the approach of the combined residuals of the orbital elements of some existing or proposed LAGEOS-type satellites, it would be possible to measure the gravitoelectric perigee advance with a relative accuracy of the order of 10^{-3} and improve the accuracy of the LARES mission, aimed to the measurement of the gravitomagnetic Lense-Thirring effect, to 1% or, perhaps, better. Moreover, the use of an entirely new pair of twin LAGEOS-type satellites placed into identical eccentric orbits with critical supplementary inclinations would allow to measure the Lense-Thirring shift not only by means of the sum of the nodes, with an accuracy of the order of less than 1%, but also, in a complementary way, by means of the difference of the perigees with an accuracy of the order of 5%.

INTRODUCTION

In the framework of the weak-field and slow-motion linearized approximation of General Relativity (GR), we will deal with some post-Newtonian effects of order $\mathcal{O}(c^{-2})$ on the orbits of test bodies freely falling in the terrestrial gravitational field and will examine recent developments about the possibility of measuring them by means of the Satellite Laser Ranging (SLR) technique.

The Gravitoelectric Effects

The geodetic, or De Sitter, precession (De Sitter, 1916) refers to the coupling of the static, gravitoelectric part of the gravitational field due to the Schwarzschild metric generated by a central, non-rotating mass to the spin of a particle freely orbiting around it. It has been measured for Earth-Moon orbit, thought of as a giant gyroscope, in the gravitational field of Sun with 1% accuracy (Williams et al., 1996). It should be measured for four superconducting gyroscopes in the gravitational field of Earth by the important GP-B mission (Everitt et al., 2001) at a claimed relative accuracy level of 2×10^{-5} . However, such mission has recently experienced some problems (Lawler, 2003a; 2003b).

Perhaps the most famous of the gravitoelectric post-Newtonian orbital effects, the advance of the pericentre ω of the orbit of a test particle (i.e., non-rotating) due to the Schwarzschild part of the metric of a central, non-rotating mass (Ciufolini and Wheeler, 1995) has represented one of the classical tests of GR thanks to the measurement of the perihelion shifts of Mercury and other planets in the gravitational field of Sun with the radar ranging technique. The relative accuracy of such measurements is of the order of 10^{-2} – 10^{-3} (Shapiro, 1990). Here we will present a proposal for obtaining a complementary measurement of such orbital effect in the gravitational field of Earth by using some existing or proposed SLR satellites of LAGEOS-type. The achievable relative accuracy should amount to 10^{-3} (Iorio et al., 2002a), according to the present-day knowledge of the models of the gravitational and non-gravitational forces acting on the satellites of LAGEOS-type.

The Gravitomagnetic Effects

Contrary to the classical Newtonian mechanics, GR predicts that the gravitational field of a central mass is sensitive to its state of motion. In particular, if it rotates and has a proper angular momentum \mathbf{J} , it turns out that a freely falling body orbiting it is affected by many so called gravitomagnetic effects. Such name is due to the close formal analogies of the linearized approximation of GR with the Maxwellian electromagnetism. For a recent review see (Ruggiero and Tartaglia, 2002). In regard to such gravitomagnetic effects, they could be divided into two main categories: those induced by the spin–spin coupling and those induced by the spin–orbit coupling.

The Gravitomagnetic Spin–Spin Effects

When a spinning particle moves in an external gravitational field one has to describe both its spin precession and the influence of the spin on its trajectory (Khriplovich, 2001).

When the spin \mathbf{J} of the central object is taken into account, it turns out that it affects the precessional motion of the spin \mathbf{s} of a particle freely orbiting it in a way discovered by Schiff (Schiff, 1960) in 1960. The detection of this subtle precessional effect, in addition to the gravitoelectric geodetic precession of the spin, is one of the most important goals of the GP-B mission (Everitt et al., 2001); the claimed accuracy amounts to 1%.

In regard to possible orbital effects on the motion of a spinning particle freely orbiting a central rotating mass in an astrophysical context, they have been recently calculated in (Iorio, 2003a). Unfortunately, they turn out to be too small to be detected in Solar System space–based experiments. See also (Zhang et al., 2001) for a different, phenomenological approach not a priori based on GR, which has been applied to some preliminary Earth–based laboratory tests (Zhou et al., 2002).

The Gravitomagnetic Spin–Orbit Effects

The Lense–Thirring drag of the inertial frames (Lense and Thirring, 1918) is an effect due to the stationary, gravitomagnetic part of the gravitational field of a central rotating mass on the geodesic path of a test particle. Such effect could be thought of as a spin–orbit interaction between the spin of the central object and the orbital angular momentum ℓ of the test body. For some spin–orbit effects induced by the rotation of Sun on the orbital angular momentum of Earth–Moon system see (Mashhoon and Theiss, 2001). In 1998 the first evidence of the Lense–Thirring effect in the gravitational field of Earth has been reported (Ciufolini et al., 1998) with a claimed accuracy of almost 20%. It is based on the analysis of a suitable combination of the orbital residuals of the nodes Ω of the LAGEOS and LAGEOS II SLR satellites and of the perigee ω of LAGEOS II.

Another interesting gravitomagnetic effect of order $\mathcal{O}(c^{-2})$ on the orbit of a test particle has been recently derived in (Iorio, 2002a); it is induced by the temporal variability of the angular momentum of the central mass. Unfortunately, it is too small to be detected with SLR in the terrestrial gravitational field.

In this paper we will show how the launch of the proposed LAGEOS–like LARES satellite could allow to measure the Lense–Thirring effect with an accuracy probably better than 1% (Iorio et al., 2002b). Moreover, the concept of twin SLR satellites placed into identical orbits in orbital planes with supplementary inclinations will be extended to new observables and a possible mission involving the use of an entirely new pair of SLR satellites of LAGEOS–type will be sketched.

THE MAJOR SYSTEMATIC ERRORS

In all the performed or proposed experiments with which we will deal in this paper the reliable assessment of the error budget is of the utmost importance. Indeed, the terrestrial space environment is rich of competing classical perturbing forces of gravitational and non–gravitational origin which in many cases are far larger than the general relativistic effects to be investigated. In particular, it is the impact of the systematic errors induced by the mismodelling in such various classical perturbations which is relevant in determining the total realistic accuracy of an experiment like those previously mentioned.

The general relativistic effects of interest here are linear trends affecting the node Ω and/or the perigee ω of the orbit of a satellite. A LAGEOS–type satellite’s orbit is affected by them at a level of 10^1 – 10^3 milliarcseconds per year (mas/y in the following) for the gravitomagnetic and the gravitoelectric effects, respectively.

In this context the most important source of systematic error is represented by the classical secular precessions of the node and the perigee induced by the mismodelled even ($l = 2n$, $n = 1, 2, 3, \dots$) zonal ($m = 0$) harmonic coefficients J_2, J_4, J_6, \dots of the multipolar expansion of the terrestrial gravitational field, called geopotential. Indeed, while the time-varying tidal orbital perturbations (Iorio, 2001; Iorio and Pavlis, 2001; Pavlis and Iorio, 2002) and non-gravitational orbital perturbations (Lucchesi, 2001; 2002), according to their periods P and to the adopted observational time span T_{obs} , can be viewed as empirically fitted quantity and can be removed from the signal, this is not the case of the classical even zonal secular precessions. Their mismodelled linear trends act as superimposed effects which may alias the recovery of the genuine general relativistic features. Such disturbing trends cannot be removed from the signal without cancelling also the general relativistic signature, so that one can only assess as more accurately as possible their impact on the measurement. The systematic error induced by the mismodelled part of the geopotential can then be viewed as a sort of unavoidable part of the total systematic error.

The same considerations hold also for the aliasing secular trends induced by some tiny non-gravitational thermal perturbations like the terrestrial Yarkovsky–Rubincam effect (Lucchesi, 2002).

The Systematic Error due to the Geopotential

A possible strategy for reducing the impact of the error due to the geopotential, as we will see in the following sections, consists of suitable combinations of the orbital residuals $\delta\dot{\Omega}$ and $\delta\dot{\omega}$ of the rates of the nodes and the perigees of different SLR satellites. Such combinations can be written in the form

$$\sum_{i=1}^N c_i f_i = X_{\text{GR}} \mu_{\text{GR}}, \quad (1)$$

in which the coefficients c_i are, in general, suitably built up with the orbital parameters of the satellites entering the combinations, the f_i are the residuals $\delta\dot{\Omega}$, $\delta\dot{\omega}$ of the rates of the nodes and the perigees of the satellites entering the combination, X_{GR} is the slope, in mas/y, of the general relativistic trend of interest and μ_{GR} is the solve-for parameter, to be determined by means of usual least-square procedures, which accounts for the general relativistic effect. For example, in the case of the Lense–Thirring–LAGEOS–LAGEOS II experiment (Ciufolini, 1996) $X_{\text{LT}} = 60.2$ mas/y, while for the gravitoelectric perigee advance (Iorio et al., 2002a) $X_{\text{GE}} = 3,348$ mas/y. More precisely, the combinations of Eq.(1) are obtained in the following way. The equations for the residuals of the rates of the N chosen orbital elements are written down, so to obtain a non homogeneous algebraic linear system of N equations in N unknowns. They are μ_{GR} and the first $N - 1$ mismodelled spherical harmonics coefficients δJ_l in terms of which the residual rates are expressed. The coefficients c_i and, consequently, X_{GR} are obtained by solving for μ_{GR} the system of equations. So, the coefficients c_i are calculated in order to cancel out the contributions of the first $N - 1$ even zonal mismodelled harmonics which represent the major source of uncertainty in the Lense–Thirring and gravitoelectric precessions (Ciufolini, 1996; Iorio, 2002b; Iorio et al., 2002a; 2002b). The coefficients c_i can be either constant¹ or depend on the orbital elements of the satellites entering the combinations.

Now we expose how to calculate the systematic error due to the mismodelled even zonal harmonics of the geopotential for the combinations involving the residuals of the nodes and the perigees of various satellites.

In general, if we have an observable q which is a function $q = q(x_j)$, $j = 1, 2, \dots, M$ of M correlated parameters x_j the error in it is given by

$$\delta q = \left[\sum_{j=1}^M \left(\frac{\partial q}{\partial x_j} \right)^2 \sigma_j^2 + 2 \sum_{h \neq k=1}^M \left(\frac{\partial q}{\partial x_h} \right) \left(\frac{\partial q}{\partial x_k} \right) \sigma_{hk}^2 \right]^{\frac{1}{2}} \quad (2)$$

in which $\sigma_j^2 \equiv C_{jj}$ and $\sigma_{hk}^2 \equiv C_{hk}$ where $\{C_{hk}\}$ is the square matrix of covariance of the parameters x_j .

In our case the observable q is any residuals' combination

$$q = \sum_{i=1}^N c_i f_i(x_j), \quad j = 1, 2, \dots, 10, \quad (3)$$

¹In general, the coefficient of the first orbital element entering a given combination is equal to 1, as for the combinations in (Ciufolini, 1996; Iorio, 2002b; Iorio et al., 2002a; 2002b).

where x_j , $j = 1, 2, \dots, 10$ are the even zonal geopotential's coefficients J_2, J_4, \dots, J_{20} . Since

$$\frac{\partial q}{\partial x_j} = \sum_{i=1}^N c_i \frac{\partial f_i}{\partial x_j}, \quad j = 1, 2, \dots, 10, \quad (4)$$

by putting Eq.(4) in Eq.(2) one obtains, in mas/y

$$\delta q = \left[\sum_{j=1}^{10} \left(\sum_{i=1}^N c_i \frac{\partial f_i}{\partial x_j} \right)^2 \sigma_j^2 + 2 \sum_{h \neq k=1}^{10} \left(\sum_{i=1}^N c_i \frac{\partial f_i}{\partial x_h} \right) \left(\sum_{i=1}^N c_i \frac{\partial f_i}{\partial x_k} \right) \sigma_{hk}^2 \right]^{\frac{1}{2}}. \quad (5)$$

The percent error, for a given general relativistic trend and for a given combination, is obtained by taking the ratio of Eq.(5) to the slope in mas/y of the general relativistic trend for the residual combination considered.

The validity of Eq.(5) has been checked by calculating with it and the covariance matrix of the EGM96 gravity model (Lemoine et al., 1998) up to degree $l = 20$ the systematic error due to the even zonal harmonics of the geopotential of the Lense–Thirring LAGEOS–LAGEOS II experiment; indeed the result

$$\delta \mu_{LT} = 13\% \mu_{LT} \quad (6)$$

claimed in (Ciufolini et al., 1998) has been obtained again. For the systematic errors due to the even zonal harmonics of the geopotential of alternative proposed gravitomagnetic and gravitoelectric experiments, according to EGM96, see (Iorio, 2002b; Iorio et al., 2002a; 2002b). It is worth noticing that, since the orbits of the LAGEOS satellites are almost insensitive to the geopotential's terms of degree higher than $l = 20$, the estimates based on the covariance matrix of EGM96 up to degree $l = 20$ should be considered rather reliable although the higher degree terms of EGM96 might be determined with a low accuracy.

It should also be pointed out that the evaluations of the systematic error due to geopotential based on the approach of the combined orbital residuals should be free from some uncertainties due to possible seasonal effects. Indeed, according to (Ries et al., 2003), the covariance matrix of EGM96 (and of other previous gravity models as well) would not yield reliable results for a generic time span of a few years, but only just for the temporal interval during which the data used for its construction have been collected or, at most, for some very long, averaged time span. It would be so because of the impact of possible secular, seasonal and stochastic variations of the terrestrial gravity field which would have not been accounted for in the model solution. However, since it turns out that such seasonal effects would mainly affect just the first even zonal harmonic coefficients of the geopotential, the uncertainty related to them should be very small for residual combinations which, by construction, cancel out just the first even zonal harmonic coefficients of the geopotential. On the other hand, if we cancel out as many even zonal harmonics as possible, the uncertainties in the evaluation of the systematic error based on the remaining correlated even zonal harmonics of higher degree should be greatly reduced, irrespectively of the chosen time span. Moreover, also certain time-dependent harmonic perturbations of gravitational origin are canceled out. I.e., the most insidious of such perturbations is the 18.6-year tide which is just a $l = 2$, $m = 0$ constituent with a period of 18.6 years due to the motion of the Moon's node and which affects both the nodes and the perigees of the LAGEOS satellites at a level of 10^3 mas (Iorio, 2001). It is cancelled out by the combinations of orbital residuals.

A very important point to stress is that the forthcoming new data on Earth gravitational field by CHAMP (Pavlis, 2000), which has been launched in July 2000, and especially GRACE (Ries et al., 2002), which has been launched in March 2002, should have a great impact on the reduction of the systematic error due to the mismodelled part of geopotential, provided that the great expectations about the level of achievable improvement of our knowledge of the terrestrial gravity field will be finally satisfied. In particular, if the other existing LAGEOS-type satellites have to be included in the combined residual combinations, it would be very important that the new terrestrial gravity models improve the accuracy in determining the even zonal harmonics of higher degree to which they are sensitive, contrary to the LAGEOS satellites. The first, very preliminary Earth gravity models including some data from CHAMP and GRACE seem to confirm such expectations (Iorio and Morea, 2003).

MEASURING THE GRAVITOELECTRIC PERIGEE ADVANCE WITH SLR

The gravitoelectric secular rate of the pericentre of a test body freely orbiting a central static mass is (Ciufolini and Wheeler, 1995)

$$\dot{\omega}_{\text{GE}} = \frac{3nGM}{c^2a(1-e^2)} \times \frac{2+2\gamma-\beta}{3}, \quad (7)$$

in which G is the Newtonian gravitational constant, c is the speed of light in vacuum, M is the mass of the central object, a and e are semimajor axis and eccentricity, respectively, of the orbit of the test body and $n = \sqrt{GM/a^3}$ is its mean motion. In the following we define $\nu = \frac{2+2\gamma-\beta}{3}$, where γ and β are the Eddington–Robertson–Schiff PPN parameters (Will, 1993) which test the alternative metric theories of gravitation. The gravitoelectric precessions for the LAGEOS satellites, according to GR ($\beta = \gamma = 1$) amount to

$$\dot{\omega}_{\text{GE}}^{\text{LAGEOS}} = 3,312.4 \text{ mas/y}, \quad \dot{\omega}_{\text{GE}}^{\text{LAGEOS II}} = 3,348.5 \text{ mas/y}. \quad (8)$$

In regard to the possibility of measuring such rates in the terrestrial gravitational field with SLR, in (Ciufolini and Matzner, 1992) a first attempt using the perigee of LAGEOS only is reported, but the estimated total systematic error is of the order of 20%.

The Use of LAGEOS and LAGEOS II

If the perigee of LAGEOS or LAGEOS II only was used, the systematic relative error due to the geopotential, according to the covariance matrix of the even zonal harmonics up to the degree $l = 20$ of EGM96, would amount to 8×10^{-3} and 2×10^{-2} , respectively. Instead, following the approach of the combined residuals outlined in the previous section it is possible to adopt as observable (Iorio 2002b; Iorio et al., 2002a)

$$\delta\dot{\omega}^{\text{LAGEOS II}} + k_1\delta\dot{\Omega}^{\text{LAGEOS II}} + k_2\delta\dot{\Omega}^{\text{LAGEOS}} = 3,348.46 \times \nu, \quad (9)$$

with

$$k_1 = -0.87, \quad k_2 = -2.86, \quad (10)$$

so that we could exploit the insight acquired with the Lense–Thirring LAGEOS–LAGEOS II experiment. The systematic relative error due to the geopotential, according to the covariance matrix of the even zonal harmonics of EGM96 up to the degree $l = 20$, amounts to 6×10^{-3} . The total systematic relative error over a time span of 8 years, including other sources of error, amounts to almost 7×10^{-3} (Iorio et al., 2002a). As a secondary outcome of such experiment, it would be possible to use its results with those for $\eta = 4\beta - \gamma - 3$ from Lunar Laser Ranging (LLR) (Anderson and Williams, 2001) in order to get independent measurements of β and γ with an accuracy of 3×10^{-3} and 1×10^{-2} , respectively. The estimates for ν , β and γ should be improved by the new forthcoming gravity models from the CHAMP and GRACE missions.

A REVISITED VERSION OF THE LARES MISSION

The Lense–Thirring secular rates of the node and the perigee of a test body freely orbiting a central rotating mass are (Lense and Thirring, 1918)

$$\dot{\Omega}_{\text{LT}} = \frac{2GJ}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \quad \dot{\omega}_{\text{LT}} = -\frac{6GJ \cos i}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \quad (11)$$

where J is the proper angular momentum of the central mass and i is the inclination of the orbit of the test particle to the equator of M . The Lense–Thirring precessions for the LAGEOS satellites amount to

$$\dot{\Omega}_{\text{LT}}^{\text{LAGEOS}} = 31 \text{ mas/y}, \quad \dot{\Omega}_{\text{LT}}^{\text{LAGEOS II}} = 31.5 \text{ mas/y}, \quad \dot{\omega}_{\text{LT}}^{\text{LAGEOS}} = 31.6 \text{ mas/y}, \quad \dot{\omega}_{\text{LT}}^{\text{LAGEOS II}} = -57 \text{ mas/y}. \quad (12)$$

The first measurement of this effect in the gravitational field of Earth has been obtained by analyzing a suitable combination of the laser-ranged data to the existing SLR LAGEOS and LAGEOS II satellites (Ciufolini et al., 1998). The observable is a linear trend with a slope of 60.2 mas/y and includes the residuals of the nodes of LAGEOS and LAGEOS II and the perigee² of LAGEOS II (Ciufolini, 1996). The claimed

²The perigee of LAGEOS was not used because it introduces large observational errors due to the smallness of the LAGEOS eccentricity (Ciufolini, 1996) which amounts to 0.0045.

total relative accuracy of the measurement of the solve-for parameter μ_{LT} , introduced in order to account for this general relativistic effect, is about 2×10^{-1} (Ciufolini et al., 1998).

As already pointed out, in this kind of experiments using Earth satellites the major source of systematic errors is represented by the aliasing trends due to the uncanceled classical secular precessions of the node and the perigee induced by the mismodelled even zonal harmonics of the geopotential J_2, J_4, J_6, \dots

In order to achieve a few percent accuracy, in (Ciufolini, 1986) it was proposed to launch a passive geodetic laser-ranged satellite- the former LAGEOS III - with the same orbital parameters of LAGEOS ($a = 12,270$ km, $e = 0.0045$, $i = 110^\circ$) apart from its inclination which should be supplementary to that of LAGEOS, i.e. $i_{LAGEOS\ III} = 70^\circ$.

This orbital configuration would be able to cancel out exactly the classical nodal precessions, which are proportional to $\cos i$ (Iorio, 2002c), provided that the observable to be adopted is the sum of the residuals of the nodal precessions of LAGEOS III and LAGEOS

$$\delta\dot{\Omega}^{LAGEOS\ III} + \delta\dot{\Omega}^{LAGEOS} = 62\mu_{LT}. \quad (13)$$

Later on the concept of the mission slightly changed. The area-to-mass ratio of LAGEOS III was reduced in order to make less relevant the impact of the non-gravitational perturbations, the total weight of the satellite was reduced to about 100 kg, i.e. to about 25% of the weight of LAGEOS, and the eccentricity was enhanced to $e_{LR} = 0.04$ in order to be able to perform other general relativistic tests: the LARES was born (Ciufolini, 1998). At present the LARES experiment has not yet been approved by any space agency. Although much cheaper than other proposed and approved complex space-based missions, funding is the major obstacle in implementing the LARES project.

Some Possible Weak Points of the Originally Proposed Lares Mission

Since the eccentricities of LAGEOS and LARES are 0.0045 and 0.04, respectively, the cancellation of the classical secular nodal precession amounts to 0.3%, according to Eq.(13) applied to the LAGEOS–LARES configuration and the covariance matrix of the even zonal harmonics of EGM96 up to the degree $l = 20$ and using the nominal values of the orbital parameters of LARES.

The major drawback of Eq.(13), applied to the LAGEOS–LARES configuration, is that it turns out to be rather sensitive to the possible departures of the LARES orbital parameters from their nominal values due to the unavoidable orbital injection errors. More precisely, for deviations of just 1° from the projected nominal supplementary configuration with LAGEOS, the systematic error due to the even zonal harmonics of the geopotential would amount to 1-2%. Of course, this fact would put rather stringent constraints on the quality and, consequently, the cost of the LARES launcher.

A New, More Accurate Residuals Combination

Following the strategy of the combined orbital residuals previously outlined, it could be possible to adopt the following observable

$$\delta\dot{\Omega}^{LAGEOS} + c_1\delta\dot{\Omega}^{LAGEOS\ II} + c_2\delta\dot{\Omega}^{LARES} + c_3\delta\dot{\omega}^{LAGEOS\ II} + c_4\delta\dot{\omega}^{LARES} = 61.8\mu_{LT}, \quad (14)$$

with

$$c_1 = 6 \times 10^{-3}, \quad c_2 = 9.83 \times 10^{-1}, \quad c_3 = -1 \times 10^{-3}, \quad c_4 = -2 \times 10^{-3}. \quad (15)$$

According to the covariance matrix of the even zonal harmonics of EGM96 up to the degree $l = 20$, the systematic error in Eq.(14) due to the geopotential reduces to 0.02%, i.e one order of magnitude better than Eq.(13). Moreover, and this is a crucial point, it turns out that such result is completely insensitive to the possible departures of the LARES orbital parameters from their nominal values, especially in regard to the inclination i_{LR} , contrary to Eq.(13). This would allow to use a rather cheap launcher for LARES. In (Iorio, 2003d) it has been shown also that the correlations among the various even zonal harmonics are not relevant for the error in Eq.(14).

In regard to the time-dependent gravitational and non-gravitational perturbations which sensibly affect the perigee, it should be noticed that the perigees of LAGEOS II and LARES are weighted, in Eq.(14), by

coefficients of order of 10^{-3} , so that the impact of the time-varying harmonic perturbations on ω would be notably reduced³.

According to the estimates of (Iorio et al., 2002b), the total systematic error should be $\leq 1\%$. Of course, when the new, hopefully more accurate data on the static and dynamical parts of the terrestrial gravitational field from the CHAMP and GRACE missions will be available, a substantial improvement in such estimate should occur.

Using LARES for the Measurement of the Gravitoelectric Perigee Advance

It would also be possible to include the data of LARES in some combinations of orbital residuals in order to measure the gravitoelectric perigee advance. E.g., a possible observable is

$$\delta\dot{\omega}^{\text{LAGEOS II}} + w_1\delta\dot{\omega}^{\text{LARES}} + w_2\delta\dot{\Omega}^{\text{LAGEOS II}} = X_{\text{GE}}\mu_{\text{GE}}, \quad (16)$$

with

$$w_1 = -4.71, \quad w_2 = 2.26, \quad X_{\text{GE}} = -12,117.4 \text{ mas/y}. \quad (17)$$

The impact of the uncanceled even zonal harmonics, with degree higher than four, amounts to an uncertainty of about 6×10^{-3} , according to the covariance matrix of the even zonal harmonics up to the degree $l = 20$ of EGM96. This error should be further reduced when the new gravity models from the CHAMP and GRACE missions will be available. An interesting feature of Eq.(16) is that the impact of the non-gravitational perturbations would produce an uncertainty of about 4×10^{-3} over a time span of 7 years, according to very conservative estimates (Iorio et al., 2002a); in particular the Earth thermal thrust, i.e. the so-called Yarkovski–Rubincam effect, would induce a mismodeled secular trend with an uncertainty of only 6×10^{-5} . For Eq.(9) the error due to the non-gravitational perturbations amounts to 1×10^{-2} , while the impact of the Yarkovski–Rubincam effect amounts to 1×10^{-4} . The inclusion of LARES would thus represent an improvement with respect to the LAGEOS and LAGEOS II scenario of Eq.(9), especially when, in the near future, the impact of the mismodeled non-gravitational perturbations will increase relatively to the gravitational perturbations thanks to the more accurate Earth gravitational field models.

NEW OBSERVABLES FOR THE SUPPLEMENTARY SATELLITES CONFIGURATION

Up to now, the concept of twin LAGEOS-type satellites in identical orbits with supplementary inclinations has been used in the context of the originally proposed LAGEOS–LARES mission whose observable is the sum of the nodes of Eq.(13).

The Difference of the Perigees

It turns out that such concept could be fruitfully extended to another independent observable sensitive to the gravitomagnetic Lense–Thirring effect. Indeed, for a pair of satellites in such orbital configuration, according to (Iorio, 2002c) and to Eq.(11), on one hand the classical secular precessions of the perigee due to the even zonal harmonics of the geopotential are equal because they depend on $\cos^2 i$ and on even powers of $\sin i$, on the other the Lense–Thirring rates of the perigee are equal and opposite because they depend on $\cos i$. So, besides the sum of the nodes, also the difference of the perigees

$$\delta\dot{\omega}^i - \delta\dot{\omega}^{180^\circ-i} = X_{\text{LT}}\mu_{\text{LT}} \quad (18)$$

could be considered as another independent, although less precise, gravitomagnetic observable (Iorio, 2003b; 2003c).

Of course, such observable could not be used in the LARES mission because the perigee of LAGEOS is not well defined. Moreover, it would neither be a good idea to think about a possible LARES II satellite supplementary to LAGEOS II because the perigee of LAGEOS II is affected by some gravitational and non-gravitational time-varying perturbations with periods of many years (Iorio, 2001; Lucchesi 2001), so that over reasonable time spans of only a few years they would resemble superimposed biasing linear trends which would corrupt the measurement of the genuine Lense–Thirring secular trend.

³This result is really important, especially in view of possible new phenomena in the surface properties of the LAGEOS satellites which might affect the perigee, as it seems to happen for LAGEOS II (Iorio et al., 2002b).

It turns out that an entirely new pair of twin LAGEOS-type satellites in identical eccentric orbits ($a = 12,000$ km, $e = 0.05$) with supplementary critical inclinations ($i^{(1)} = 63.4^\circ$, $i^{(2)} = 116.6^\circ$) would be a good choice. Indeed, the so obtained frozen perigee configuration⁴ would allow to reduce the periods of all the time-dependent harmonic perturbations, so that they could be easily fitted and removed from the signal over a time span of a few years. Moreover, since the eccentricities would be the same, the cancellation of the mismodelled classical secular precessions due to the even zonal harmonics of the geopotential would occur at an higher level of accuracy than that of the originally proposed LAGEOS-LARES mission.

According to numerical estimates based on the physical and geometrical properties of the existing LAGEOS satellites, the total accuracy over an observational time span of 6 years would be of the order of 5% (Iorio and Lucchesi, 2003). In it a very important role would be played by the non-gravitational perturbations. It is worth noticing that their importance in the determination of the total error budget will greatly increase when the new, more accurate data for the static and time-varying parts of Earth gravitational field will be available from the CHAMP and, especially, GRACE missions.

Of course, such new configuration would allow to increase the precision of the sum of the nodes as well. According to very preliminary numerical estimates based on the physical and geometrical properties of the LAGEOS satellites, the total accuracy over an observational time span of 6 years would be $\leq 1\%$ (Iorio and Lucchesi, 2003).

The addition of the sum of the nodes and of the difference of the perigees

$$\delta\dot{\Omega}^i + \delta\dot{\Omega}^{180^\circ-i} + \delta\dot{\omega}^i - \delta\dot{\omega}^{180^\circ-i} = X_{LT}\mu_{LT} \quad (19)$$

would increase the accuracy to 2.8% (Iorio and Lucchesi, 2003). Such estimates should further improve because they do not account for the possibility of fitting and removing some time-dependent orbital perturbations with known periodicities from the time series.

The Combined Residuals Scenario

If, instead of LARES, we would use the data of one of the two new proposed satellites with critical inclination in some combinations of orbital residuals we could measure both the gravitoelectric perigee advance and the Lense-Thirring effect. A possible observable would be

$$\delta\dot{\omega}^i + p_1\delta\dot{\Omega}^{\text{LAGEOS II}} + p_2\delta\dot{\Omega}^{\text{LAGEOS}} + p_3\delta\dot{\omega}^{\text{LAGEOS II}} + p_4\delta\dot{\Omega}^i = \mu_{GR}X_{GR}, \quad (20)$$

in which

$$p_1 = -1.55, \quad p_2 = -2.77, \quad p_3 = 0.348, \quad p_4 = 0.361. \quad (21)$$

For the gravitoelectric perigee advance we would have $X_{GE} = 4,636.5$ mas/y and a systematic relative error due to the geopotential of 2×10^{-4} , according to the covariance matrix of the even zonal harmonics of EGM96 up to the degree $l = 20$. For the gravitomagnetic Lense-Thirring effect we would have $X_{LT} = -187$ mas/y and a systematic relative error due to the geopotential of 5×10^{-3} , according to the covariance matrix of the even zonal harmonics of EGM96 up to the degree $l = 20$. In regard to the measurement of the Lense-Thirring effect, the combination of Eq.(14) involving LARES, LAGEOS and LAGEOS II would be more accurate than Eq.(20). On the other hand, in regard to the measurement of the gravitoelectric perigee advance, Eq.(20) would represent a notable improvement with respect to Eq.(16).

CONCLUSIONS

In this paper we have illustrated some new recent developments in the field of precise testing GR with SLR.

By suitably combining the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II it would be possible to obtain a complementary measurement of the gravitoelectric perigee advance in the gravitational field of Earth with the SLR technique at an accuracy level of 0.7%.

A modified version of the observable to be used in the LARES mission, which includes not only the node of LAGEOS but also the node and the perigee of LAGEOS II and the perigee of LARES in a suitable combination, should be able to measure the gravitomagnetic Lense-Thirring effect at an accuracy level of

⁴For $i = 63.4^\circ$ the classical secular precession on ω due to J_2 vanishes.

the order of 1% by using a not too expensive launcher for LARES. Indeed, the error due to the even zonal harmonics of the new proposed observable would be insensitive to the unavoidable orbital injection errors, to the correlations among the even zonal harmonic coefficients and, to a certain extent, to the adopted Earth gravity model.

The use of a couple of entirely new SLR LAGEOS-type satellites placed into identical eccentric orbits with critical supplementary inclinations would allow to use not only the sum of the nodes but also the difference of the perigees as independent observables for the measurement of the Lense–Thirring effect. According to preliminary numerical estimates based on the properties of the existing LAGEOS satellites, the accuracy would be of the order of 1% and 5%, respectively.

When the new, more accurate data for the terrestrial gravitational field from the CHAMP and, especially, GRACE missions will be available, such estimates should be notably improved. The first, very preliminary, Earth gravity models including some data of CHAMP and GRACE seem to confirm such expectations.

REFERENCES

- Anderson, J.D., and J.G. Williams, Long-range tests of the equivalence principle, *Class. and Quantum Grav.*, **18**(13), 2447–2456, 2001.
- Ciufolini, I., Measurement of Lense–Thirring drag on high-altitude laser ranged artificial satellite, *Phys. Rev. Lett.*, **56**(4), 278–281, 1986.
- Ciufolini, I., On a new method to measure the gravitomagnetic field using two orbiting satellites, *Il Nuovo Cimento A*, **109**(12), 1709–1720, 1996.
- Ciufolini, I., The Concept of the LARES experiment, in *LARES Phase–A Study*, pp. 16–33, Rome, 1998.
- Ciufolini, I., and R. Matzner, Non-Riemannian theories of gravity and lunar and satellite laser ranging, *Int. J. of Mod. Phys. A*, **7**(4), 843–852, 1992.
- Ciufolini, I., and J.A. Wheeler, *Gravitation and Inertia*, Princeton University Press, New York, 1995.
- Ciufolini, I., E. Pavlis, F. Chieppa, et al., Test of General Relativity and Measurement of the Lense–Thirring Effect with Two Earth Satellites, *Science*, **279**, 2100–2103, 1998.
- De Sitter, W., On Einstein’s theory of gravitation and its astronomical consequences, *Mon. Not. Roy. Astron. Soc.* **77**, 155–184 and 481, 1916.
- Everitt, C.W.F., S. Buchman, D.B. DeBra, and other members of the Gravity Probe B Team, Gravity Probe B: Countdown to Launch, in *Gyros, Clocks, Interferometers...:Testing Relativistic Gravity in Space*, ed. Lämmerzahl, C., C.W.F. Everitt, and F.W. Hehl, pp. 52–82, Springer Verlag, Berlin, 2001.
- Iorio, L., Earth tides and Lense–Thirring effect, *Celest. Mech. & Dyn. Astron.*, **79**(3), 201–230, 2001.
- Iorio, L., A Gravitomagnetic Effect on the Orbit of a Test Body due to the Earth’s Variable Angular Momentum, *Int. J. of Mod. Phys. D*, **11**(5), 781–787, 2002a.
- Iorio, L., Is it possible to improve the present LAGEOS–LAGEOS II Lense–Thirring experiment? , *Class. and Quantum Gravity*, **19**(21), 5473–5480, 2002b.
- Iorio, L., Recent Developments in Testing General Relativity with Satellite Laser Ranging, *La Rivista del Nuovo Cimento*, **25**(5), 2002c.
- Iorio, L., On Some Gravitomagnetic Spin-Spin Effects for Astronomical Bodies, *Int. J. of Mod. Phys. D*, **12**(1), 35–44, 2003a.
- Iorio, L., A new proposal for measuring the Lense–Thirring effect with a pair of supplementary satellites in the gravitational field of the Earth, *Phys. Lett. A*, **308**(2–3), 81–84, 2003b.
- Iorio, L., On a new observable for measuring the Lense–Thirring effect with Satellite Laser Ranging, *Gen. Rel. Grav.*, **35**(9), 1583–1595, 2003c.
- Iorio, L., A reassessment of the systematic gravitational error in the LARES mission, *Gen. Rel. Grav.*, **35**(7), 1263–1272, 2003d.
- Iorio, L. and E.C. Pavlis, Tidal Satellite Perturbations and the Lense–Thirring Effect, *J. of the Geod. Soc. of Japan*, **47**(1), 169–173, 2001.
- Iorio, L. and D.M. Lucchesi, LAGEOS-type Satellites in Critical Supplementary Orbital Configuration and the Lense–Thirring Effect Detection, *Class. and Quantum Gravity* **20**(13), 2477–2490, 2003.
- Iorio, L. and A. Morea, The impact of the new Earth gravity models on the measurement of the Lense–Thirring effect, <http://www.arxiv.org/abs/gr-qc/0304011>, 2003c.

- Iorio, L., I. Ciufolini, and E.C. Pavlis, Measuring the Relativistic Perigee Advance with Satellite Laser Ranging, *Class. and Quantum Grav.*, **19**(16), 4301-4309, 2002a.
- Iorio, L., D.M. Lucchesi, and I. Ciufolini, The LARES Mission Revisited: An Alternative Scenario, *Class. and Quantum Grav.*, **19**(16), 4311-4325, 2002b.
- Khriplovich, I.B., Spinning Relativistic Particles in External Fields, in *Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space*, ed. Lämmerzahl, C., C.W.F. Everitt, and F.W. Hehl, pp. 109-130, Springer Verlag, Berlin, 2001.
- Lawler, A., Gravity Mission Faces Make-or-Break Review *Science*, **299**(5614) 1827-1829, 2003a.
- Lawler, A., NASA Orders Make-or-Break Tests for Gravity Probe *Science* **300**(5621) 880, 2003b.
- Lemoine, F.G., S.C. Kenyon, J.K. Factor, et al., The Development of the Joint NASA GSFC and the National Imagery Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP-1998-206861, 1998.
- Lense, J., and H. Thirring, Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, *Phys. Z.*, **19**, 156-163, 1918, translated by Mashhoon, B., F. W. Hehl, and D. S. Theiss, On the Gravitational Effects of Rotating Masses: The Thirring-Lense Papers, *Gen. Rel. Grav.*, **16**, 711-750, 1984.
- Lucchesi, D.M., Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring determination. Part I, *Plan. and Space Sci.*, **49**(5), 447-463, 2001.
- Lucchesi, D.M., Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring determination. Part II, *Plan. and Space Sci.*, **50**(10-11), 1067-1100, 2002.
- Mashhoon, B., and D.S. Theiss, Relativistic Effects in the Motion of the Moon, in *Gyros, Clocks, Interferometers...: Testing Relativistic Gravity in Space*, ed. Lämmerzahl, C., C.W.F. Everitt, and F.W. Hehl, pp. 310-316, Springer Verlag, Berlin, 2001.
- Pavlis, E.C., Geodetic Contributions to Gravitational Experiments in Space, *Recent Developments in General Relativity*, ed. R. Cianci, R. Collina, M. Francaviglia, and P. Fré, pp. 217-233, Springer-Verlag, Milan, 2000.
- Pavlis, E.C., and L. Iorio, The impact of tidal errors on the determination of the Lense-Thirring effect from satellite laser ranging, *Int. J. of Mod. Phys. D*, **11**(4), 599-618, 2002.
- Ries, J. C., R.J. Eanes, B.D. Tapley, and G.E. Peterson, Prospects for an Improved Lense-Thirring Test with SLR and the GRACE Gravity Mission, in *Proceedings of the 13th International Laser Ranging Workshop*, Washington DC, October 7-11, 2002, http://cddisa.gsfc.nasa.gov/lw13/lw_proceedings.html#science, 2002.
- Ries, J.C., R.J. Eanes, and B.D. Tapley, Lense-Thirring Precession Determination from Laser Ranging to Artificial Satellites, *Nonlinear Gravitodynamics. The Lense-Thirring Effect*, ed. R. Ruffini, and C. Sigismondi, pp. 201-211, World Scientific, Singapore, 2003.
- Ruggiero, M.L., and A. Tartaglia, Gravitomagnetic Effects, *Il Nuovo Cimento B*, **117**(7), 743-767, 2002.
- Schiff, L.I., On Experimental Tests of the General Theory of Relativity, *Am. J. Phys.*, **28**(4), 340-343, 1960.
- Shapiro, I., Solar system tests of general relativity: recent results and present plans, in *Proceedings of the 12th International Conference on General Relativity and Gravitation*, ed. N. Ashby, D. Bartlett, and W. Wyss, pp. 313-330, Cambridge University Press, Cambridge, 1990.
- Will, C M., *Theory and Experiment in Gravitational Physics*, 2nd edition, Cambridge University Press, Cambridge, 1993.
- Williams, J.G., X.X. Newhall, and J.O. Dickey, Relativity parameters determined from lunar laser ranging, *Phys. Rev. D* **53**(12), 6730-6739, 1996.
- Zhang, Y.Z., J. Luo, and Y.X. Nie, Gravitational Effects of Rotating Bodies, *Mod. Phys. Lett. A*, **16**(12), 789-794, 2001.
- Zhou, Z.B., J. Luo, Q. Yan, Z.G. Wu, Y.Z. Zhang, and Y.X. Nie, New upper limit from terrestrial equivalence principle test for extended rotating bodies, *Phys. Rev. D*, **66**(2), 022002, 2002.

E-mail address of L.Iorio Lorenzo.Iorio@ba.infn.it

Manuscript received 19 October 2002